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# Chaos Theory Applied to the Caloric Response of the Vestibular System

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Developments in the field of nonlinear dynamics has given us a new conceptual framework for understanding the mechanisms involved in the regulation of complex nonlinear systems. This concept, called "chaos" or "deterministic chaos," has been applied to EKG, EEG, and other physiological signals, but not yet to the ENG signal. The underlying geometrical structure in chaotic dynamics is fractal (noninteger dimension), and calculating the fractal dimension of the electronystagmographic recording from caloric testing gave a dimension ranging from 3.3 to 7.7. This result demonstrates that the multidimensional vestibular system, with its numerous neurological pathways, can somehow reduce the degrees of freedom and give rise to an irregular dynamic low-dimensional behavior, which is associated with deterministic chaos. © 1993 Academic Press, Inc.

## 1. INTRODUCTION

The main purposes of this paper are (a) to present the idea of nonlinear dynamics applied to the vestibular system—which is one of the three sensory systems concerned with balance and equilibrium (in addition to the visual and the somatosensory system)—and (b) to suggest using the fractal dimension as a parameter to quantify the irregular pattern of the nystagmus movements.

Nystagmus is a rhythmic, involuntary, back-and-forth eye movement. The electrical recording of nystagmus is called electronystagmography (ENG). ENG is possible because the eye is charged positively at the cornea and negatively at the retina, creating an electrical dipole known as the corneoretinal potential (*1*).

Considerations about the nystagmus dynamics presented here are based on a time-series analysis of ENG recordings from standard clinical caloric testing. Caloric nystagmus is produced by irrigating the external auditory canal with water 7°C below and above body temperature (Fig. 1). The caloric test determines if the lateral semicircular canal and its nervous pathways are functioning normally.

The experimental approach is inspired by ideas from the field of nonlinear dynamics which is called deterministic chaos.



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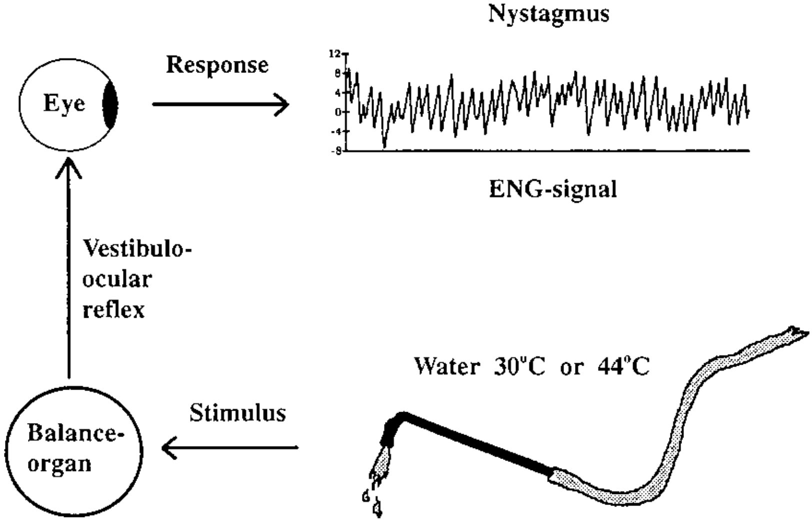


FIG. 1. The caloric test determines if the labyrinthine end organs and the nervous system pathways are functioning normally. Cold or warm water is irrigated into the external auditory canal, and the resulting eye movements are recorded for analysis.

The dynamical interpretation of the vestibular system through the ENG signal is from the following assumptions:

(1) The nystagmus dynamics during caloric testing reflect intrinsic dynamic properties of the regulating mechanism in the underlying vestibular control system.

(2) Information about the control mechanism governing the nystagmus response is hidden in the irregular pattern of the ENG signal.

(3) Nonlinear dynamic properties of the vestibular system generate a complex irregular nystagmus movement which is there on purpose—i.e., as a vital and necessary regulating function.

### *1.1. Irregularity as Noise and Irregularity as Intrinsic Dynamic Properties*

It is important to emphasize the difference between (a) irregularity as randomly generated variations and (b) irregularity/chaos determined by intrinsic dynamical properties of the physiological regulating system:

(a) Irregularity as a result of increased noise; the physiological system does not regulate, but functions more like a noise generator.

(b) Irregularity determined by intrinsic dynamic properties in the physiological regulating system for the purpose of inhibiting intense stimuli and providing the system with a flexible and functional regulatory mechanism.

### *1.2. Irregularity—A Hallmark of Physiological Healthiness?*

A system which responds proportionally to a disturbance is mathematically easy to interpret. The mechanisms governing the human physiology are more complex. It is hypothesized that the normal irregular nystagmus behavior (Fig.

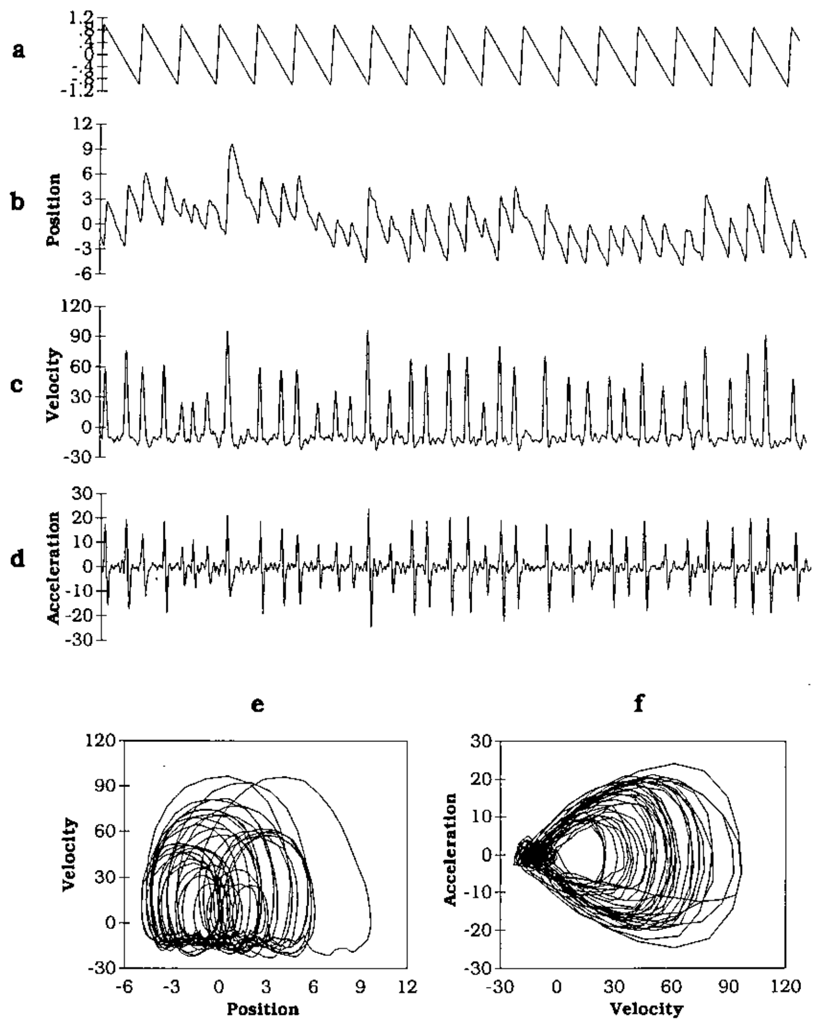


FIG. 2. (a) The expression of "ideal nystagmus" is a highly theoretical notion which refers to a regular and predictable nystagmus. (b) A real caloric-induced nystagmus is in contrast a back-and-forth eye movement which never repeats itself, a complex behavior where it is impossible to predict when the slow phase stops and the fast phase begins, and vice versa. (Upward direction represents eye movement to the right, and downward direction represents eye movement to the left.) (c) The eye velocity (derivative of (b)). (d) The eye acceleration (derivative of (c)). (e) Topographic representation of the nystagmus dynamics in the phase plane (position and velocity). (f) Phase-plane plot of acceleration versus velocity. (All plots are given for a 20-sec time period.)

2) is a result of dynamic properties in the vestibular and eye movement system whose purpose is to maintain control. Alan Garfinkel illustratively describes the catastrophical mechanism of too regular behavior in "Open Peer Commentary" to Skarda and Freeman's article, "How Brain Makes Chaos in Order to Make Sense of the World" (*2*):

> "In epileptic seizures, the EEG becomes regular and periodic, and it is the normal ("desynchronized") EEG that is irregular. Given the undesirability of periodic cortical behavior, it is reasonable to suppose that the nervous system has evolved a reliable mechanism to desynchronize the EEG. As an example of the utility of such "active desynchronization", consider the behavior of a platoon of soldiers crossing a bridge. Since periodic behavior (marching in ranks) might set the bridge into destructive resonant oscillation, the soldiers "break ranks". . . . In general, it may be that for all oscillatory processes in physiology, a perfect periodic oscillation is undesirable. Chaos could here play the role of introducing a useful wobble into the period or amplitude, while retaining the overall form of the process."

In 1982 Fluur reported from a study of rotatational testing of dizzy patients (*3*):

> ". . . efferent inhibition . . .modulates the nystagmus as a kind of defence mechanism against too intensive stimuli . . . the nystagmus becomes irregular when the inhibition increases but, on the other hand, becomes more regular when the inhibition disappears."

Although Fluur describes irregular nystagmus behavior in contrast to the "generally more regular nystagmus," he expresses the same idea of a mechanism which provides "controlled randomness" for the purpose of maintaining control.

### *1.3. Deterministic Chaos*

Recent developments in the field of nonlinear dynamics have led to new ways of understanding the mechanism involved in regulating complex nonlinear systems. This new concept is called *chaos* or *deterministic chaos* (*4–7*), and it refers to irregularity that arises in a deterministic system. Chaotic systems exhibit *sensitive dependence on initial conditions*, which means that the system balances on the boundary between multiple possibilities where a slight perturbation forces the system in a new direction. Because of the system's sensitive dependence on the environment, long-term prediction becomes impossible.

A characteristic property of chaotic dynamics is that the underlying structure of the complex geometric shape in phase space (see below) has an inner regularity. This regularity can be calculated, and the measure is called the fractal dimension. The term fractal was coined by Benoit B. Mandelbrot (*8*).

### *1.4. Fractal Geometry*

In the terminology of classical geometry we live in a three-dimensional space, the surface has two dimensions, and the line has one dimension. Fractal geometry, on the other hand, is recognized by *noninteger dimensions*. A coastline, for instance, will have a dimension between one and two (a higher dimension than a one-dimensional line and a lower dimension than a two-dimensional surface). Another characteristic property of fractal geometry is that the geometric structure is *self-similar* across different scales. It is not identical in details, but the structure will essentially have the same statistical properties. If you, for example, zoom in on the branches of a tree, new branches become visible. At higher magnification more details are unveiled. Physically there exists a lower boundary for this branching structure. Idealized fractals, i.e., fractals governed by mathematical equations, have infinite branching patterns or structures.

Anatomical fractal-like structures can be found in the airways of the lung and in the networks of blood vessels, nerves, and ducts (*9*).

Fractals can also be illustrated by a definition of dimension, which calculates the regularity inherent in the chaotic structure of a *strange attractor* (see below) or the branching pattern of a geometric shape. One can think of the fractal dimension both as a measure of the amount of space which is occupied and as a scaling factor which accounts for the self-similar structure.

There are many measures of the dimension of a fractal structure. The most common definition of dimension is *capacity dimension*. Another definition of fractal dimension, which has been applied successfully on time-series data, is *correlation dimension* (*6, 10, 11*).

The fractal dimension of a time-series is also related to the smallest number of first-order differential equations necessary to capture the qualitative features of the system's dynamics.

### *1.5. Phase Space and Attractors*

A system's dynamical behavior can be represented in an abstract mathematical space known as phase space. It takes the form of a trajectory which reflects the system's behavior over a given period of time. The axis may consist of a combination of the position versus velocity versus acceleration, or some other active variables which determine the system's evolvement in time. The subset of points in phase space which trajectories return to after all transients die out is referred to as an *attractor*. Figure 3 shows a fixed-point attractor (IIIa), a limit cycle (IIIb), and a strange (chaotic) attractor (IIIc). The nystagmus attractor (nystagmus spectrum) reflects a complicated dynamic behavior where nonlinear properties spread the trajectories to a fan-like structure and then fold it back to a small local area of space. The procedure of stretching and folding prevents predictability of the system's history, without destroying the global structure. The phase space plot of IIId (Fig. 3) takes the form of a randomly occupied area with no global structure. Figure 4 shows a three-dimensional portrait (position–velocity–acceleration) of the nystagmus attractor.

## 2. Methods

### *2.1. Material and Recording Techniques*

The ENG signals were recorded in three healthy subjects, according to conventional caloric test procedures, as introduced by Fitzgerald and Hallpike in 1942 (*12*). Caloric nystagmus was obtained by running water 7°C below and

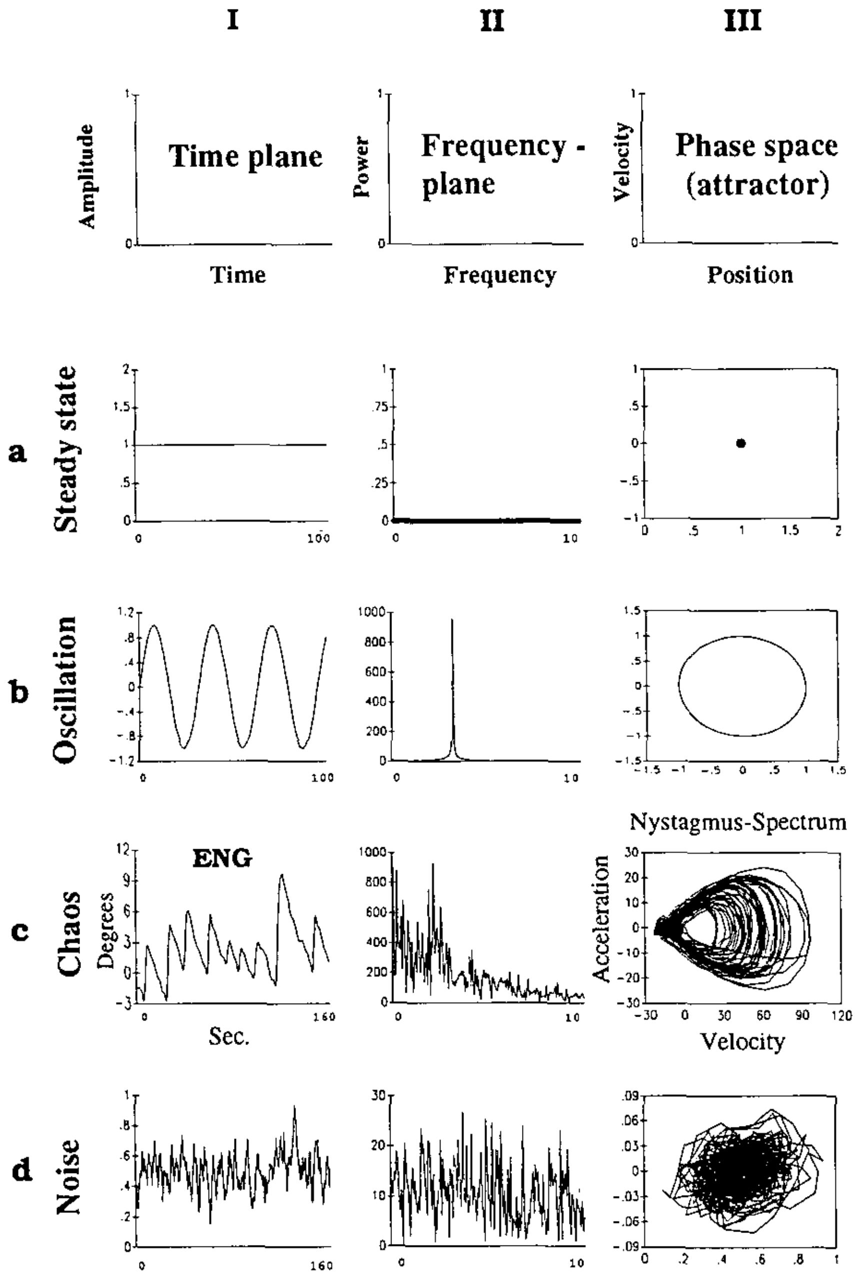


FIG. 3. Four main mathematical models have been developed to characterize time-series (7): (a) steady states, (b) oscillations, (c) chaos, and (d) noise. (I) time plane, (II) frequency plane, (III) phase space.

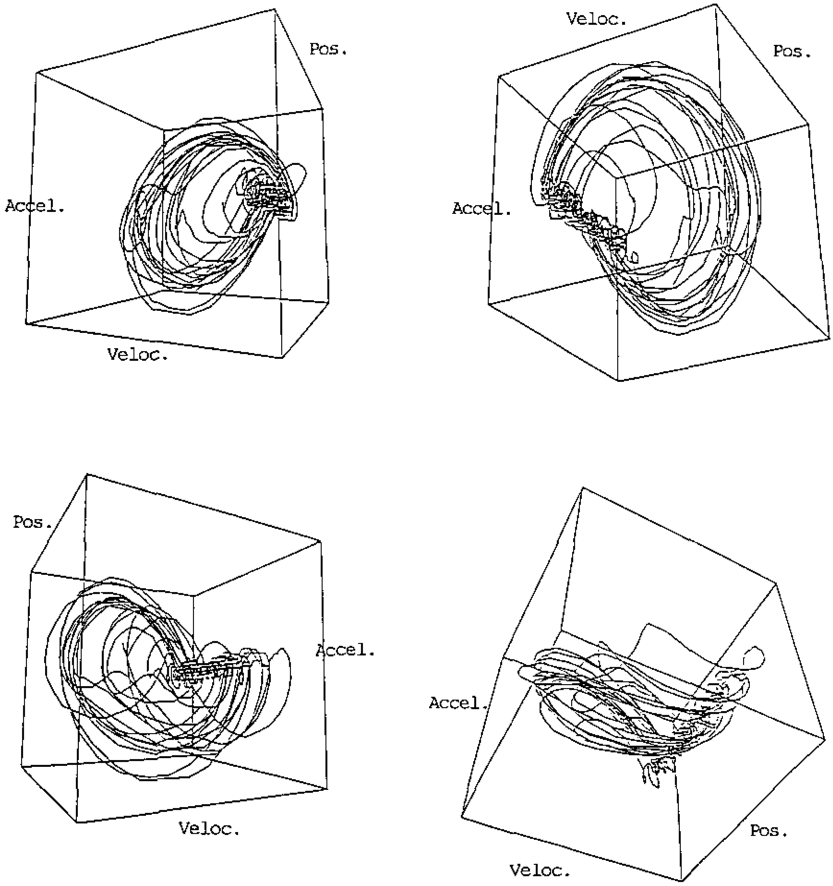


FIG. 4. The nystagmus attractor presented in a three-dimensional phase space portrait (position–velocity–acceleration). The region where the trajectories follow close to each other represents nystagmus' slow component. The attractor is presented from four different viewpoints.

above body temperature into the external auditory canals. Horizontal eye movements were recorded with two electrodes placed at each side of the eyes and a reference electrode in the center of the forehead. The data were digitized into an IBM-compatible computer, using 12-bit A/D resolution and 100- or 200-Hz sampling frequency.

### 2.2. *Smoothing the Data*

A nonlinear algorithm, called the *simplified least-squares procedure,* was used for smoothing the recorded ENG signals (*13*).

A polynomial of degree $n < 2m + 1$ was fitted to a set of $2m + 1$ data points in such a way that the mean square of the difference between calculated values and the time-series values was a minimum. The center abscissa of the set was then substituted with the central point of the polynom, which is the best mean square fit. The point at one end of the data set was then dropped, and the next point at the other end was added. The process was repeated until the whole time-series of data points had been modified.

### 2.3. *Pseudo-Phase Space*

One way to reconstruct the complex behavior of a system consisting of many interacting components from a single time variable is by introducing a time delay (*10, 11, 14*). Changes in one variable will be recognized by other variables, and the interaction of the components will determine the system's dynamics.

The time-series

$$X_0\colon X_0(t_1), X_0(t_2), \ldots, X_0(t_N),$$

where $N$ is the number of data points, can be reconstructed in a multidimensional phase space.

If the number of variables is defined as $n$, we get

$$\begin{aligned} &X_0\colon X_0(t_1), X_0(t_2), \ldots, X_0(t_N) \\ &X_1\colon X_0(t_1+\tau), X_0(t_2+\tau), \ldots, X_0(t_N+\tau) \\ &\quad\cdot \\ &\quad\cdot \\ &X_{n-1}\colon X_0(t_1+(n-1)\tau), \ldots, X_0(t_N+(n-1)\tau), \end{aligned}$$

where $\tau = m\Delta t$ is the time delay, $m$ is an integer, and $\Delta t$ is the sampling interval.

The time delay must neither be too small, because of the linear dependency $X_0 \approx X_1 \approx X_2 \approx \cdots \approx X_{n-1}$, nor be too large, since that will result in loss of information (*11*).

### 2.4. *The Correlation Dimension*

Computing the correlation dimension from a single time-series involves calculating the distances between pairs of points, $|X_i - X_j|$, in pseudo-phase space, and then counting the data points within a distance, $r$, from point $X_i$. A multidimensional correlation integral $C(r)$ is defined as

$$C(r) = \frac{1}{N^2}\sum_{i,j=1}^{N} \theta(r - |X_i - X_j|),$$

where $\theta$ is the Heaviside function

$$\theta(x) = 0 \quad \text{if } x < 0 \qquad \text{and} \qquad \theta(x) = 1 \quad \text{if } x \geq 0.$$

The correlation dimension $d_g$ is the relation between log $C(r)$ and log $r$

$$d_g = \lim_{r\to 0} \frac{\log C(r)}{\log r}.$$

See Refs. (*6, 10, 11*).

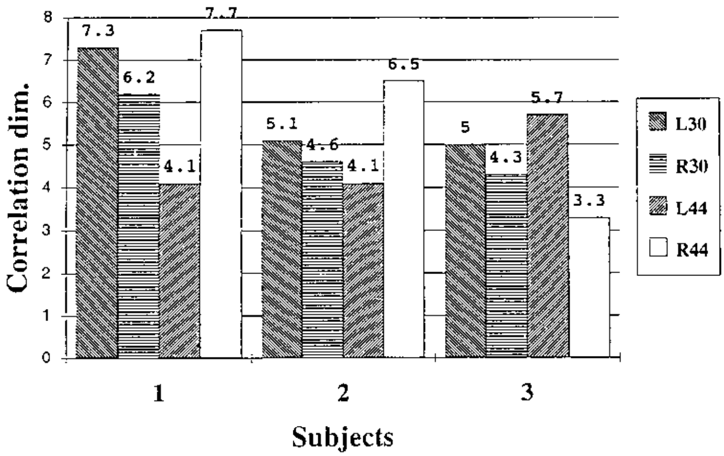


FIG. 5. The correlation dimension calculated for nystagmus signals recorded after 30 sec irrigation of left ear with water at 30°C (L30), 30 sec irrigation of right ear with water at 30°C (R30), 30 sec irrigation of left ear with water at 44°C (L44), and 30 sec irrigation of right ear with water at 44°C (R44).

## 3. RESULTS

Figure 3a shows that the steady state has no dynamics—no geometry in phase space. The fractal dimension of the steady state is zero. Oscillation (Fig. 3b) is a predictable ever-repeating behavior, with a fractal dimension of one (one degree of freedom). Noise (Fig. 3d) has no underlying deterministic structuring components, and randomly occupies the phase plane. The dimension for a so-called white noise signal is infinite. Deterministic chaos, illustrated with an ENG signal (Fig. 3c), has a global well-defined and stable phase portrait, but is unpredictable and unstable in the local regions.

### *3.1. The dimension of the Irregular Nystagmus Pattern*

The fractal dimension of the ENG signals was calculated according to the correlation dimension method introduced by Grassberger and Procaccia (*10*, *11*), briefly described above. Approximately 2000 data points ($N + (n - 1)\tau$) from the period with the strongest nystagmus response were selected from the electronystagmographic signals. The selecting criterion was the maximum slow-phase velocity (SPV), calculated in degrees/sec.

The time-series was reconstructed in a pseudo-phase space, with a time delay, $\tau$, corresponding to the first zero crossing, or the first minimim of the autocorrelation function. The correlation dimension (the slope of $\log C(r)$ versus $\log r$ in the linear region) was calculated for increasingly higher-dimensional pseudo-phase spaces, until the dimension value reached an asymptote. This asymptotic value was considered an estimate of the fractal dimension.

The correlation dimension calculated ranged from 3.3 to 7.7, and is summarized in Fig. 5.

The correlation dimension measures the ENG signal, projected as a geometric object in a multidimensional pseudo-phase space. This geometric object is highly sensitive to dynamic variations in the underlying physiological system and is therefore of great value in determining the system's complexity. The results presented in this paper demonstrate that the high-dimensional vestibular system, with its numerous neurological pathways, can somehow reduce the degrees of freedom and give rise to a dynamic low-dimensional, optimal function. (The main function of the vestibulo-ocular reflex is to hold images steady on the retina during head movements). Low-dimensional, irregular time-dependent behavior is a property associated with deterministic chaos.

## 4. Discussion

Huberman (*15*) has presented a mathematical model for dysfunctions in smooth pursuit eye movement, which exhibits chaotic behavior. His paper focuses on dysfunction—the eyes' inability to smoothly track a periodically moving target—as a phenomenon arising from nonlinearities in the eye tracking mechanisms through deterministic chaos.

The present paper questions if irregularities of the ENG signal from caloric stimulation reflect the system's adaptability to the multifunctioning physiological environment. There must be a dynamic regime where the eye motor control system serves the requirement of vision. In addition there may exist a regime where nonlinear properties/chaos in the vestibular and eye motor control system autonomously regulate the system as a defense mechanism against too strong stimuli. If the stimulus threshold for driving the system into chaos is too low, irregular behavior can impair the eyes' ability to focus on a moving target. Thus, the onset of chaos could be a diagnostic criterion. If the two regimes exist, it will be an important goal to identify the boundary between them. This will necessitate identifying the physiological parameters which determine the dynamic behavior of the two states and exploring the intensity of the stimuli that forces the transition between nonchaotic and chaotic behavior.

Calculating the correlation dimension is a complicated procedure, where dynamical stationarity is presumed over a minimum period of time. It requires a set of data points large enough to cover the main structure of the attractor. However, the caloric reaction is a temporary response which increases to a maximum, and then decreases. After 2–3 min the response fades out. It can be difficult to find a time-series of sufficient duration to get a reliable measure of the fractal dimension. It must also be emphasized that the values given for the fractal dimension of the ENG signal in this paper should be regarded as estimates, a first attempt to apply the idea of fractals to vestibular testing. A standardized methodology for calculating the dimension (choosing an appropriate delay time, identifying a linear scaling region, etc.) does not exist. Further efforts to apply fractal analysis in the field of vestibular testing will require studies on the many programming-technical problems involved with calculation of the dimension. The otoneurological implications of the findings must be discussed for a physiological explanation, and the possible clinical benefit will be a result of extensive studies on the different groups of patients tested in the vestibular laboratory.

From the above discussion of the nystagmus dynamics, a fractal diagnostic hypothesis of disturbances in the vestibular control system can be put forward: Disturbances of the verstibular system can change the correlation dimension in two ways: (1) a pathologically decreased dimension because of reduced variability, and thus a reduced ability to rapidly regulate the system, and (2) a pathologically increased dimension because of increased noise—the physiological system looses its regulatory functions and is acting more like a noise generator.

The above distinction motivates the development of techniques for quantifying irregular nystagmus patterns for diagnostic purposes.

## 4. Conclusion

The caloric reaction was discovered around 1850 (*16*). One and a half centuries later, the triggering mechanisms of the nystagmus' fast phase (the active phase, from a dynamical point of view), and the underlying control function of the beat-to-beat variations of the nystagmus, are still not understood.

The chaos model offers a new theoretical framework for understanding the regulating mechanisms concerned with balance and equilibrium. This framework is oriented towards *biological information processing* (the system's behavior is determined by information from the vestibular, the visual, and the somatosensory system), *adaptability* (to the multifunctioning physiological and external environment), and *optimizing* (to orient the body in space and to hold images steady on the retina). The complex unpredictable beating of nystagmus is interpreted as a phenomenon accompanying a healthy functional system.

In the vestibular laboratory, various techniques are available for a routine clinical examination. Developing new methods to distinguish between pathological and normal variations in the ENG signal may expand the vestibular test battery and improve ENG as a diagnostic tool. The development of chaos theory and fractal analysis might be a milestone toward reaching this goal.

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